International Journal of Theoretical Physics, Vol. 36, No. 7, 1997

Physical Versus Computational Complementarity. I

Cristian Calude,[1] Elena Calude,[2] Karl Svozil,[3] and Sheng Yu[4]

Received September 3, 1996

The dichotomy between endophysical/intrinsic and exophysical/extrinsic perception concerns how a model—mathematical, logical, computational—universe is perceived, from inside or from outside. This paper, the first in a proposed series, discusses some limitations and tradeoffs between endophysical/intrinsic and exophysical/extrinsic perceptions in both physical and computational contexts. We build our work on E. F. Moore's Gedanken-experiments in which the universe is modeled by a finite deterministic automaton. A new type of computational complementarity, which mimics the state of quantum entanglement, is introduced and contrasted with Moore's computational complementarity. Computer simulations of both types of computational complementarity are developed for four-state Moore automata.

1. INTRODUCTION

This section is mainly expository: we will present the physical and mathematical contexts for our work.

1.1. Physical Complementarity

Relativity altered the classical concept of physical objectivity, but left open the possibility of a supreme mathematician who, in Einstein's view, neither cheats nor plays dice. Quantum mechanics goes one step further: the experimenter can neither predict nor control certain "spontaneous" micro-

[1] Computer Science Department, University of Auckland, Auckland, New Zealand; e-mail: cristian@cs.auckland.ac.nz.
[2] Computer Science Department, University of Auckland, Auckland, New Zealand; e-mail: elena@cs.auckland.ac.nz.
[3] Institut für Theoretische Physik, University of Technology Vienna, A-1040 Vienna, Austria; e-mail: svozil@tph.tuwien.ac.at.
[4] Department of Computer Science, University of Western Ontario, London, Ontario, Canada N6A 5B7; e-mail: syu@csd.uwo.ca.

0020-7748/97/0700-1495$12.50/0

physical events. Moreover, the observer is bound by complementarity—that is, informally speaking, either experiences a particular type of observation (exclusive) or a different, complementary one.

Physical complementarity appears to be a rather straightforward consequence of the quantum formalism. Yet the conceptualization of complementarity has caused considerable attention, concern, thought, and neglect (Jammer, 1966, 1974). Indeed, physical complementarity is tied up with measurement. The notion of measurement, in turn, is a highly nontrivial matter, as contemplations by Wigner (1961), Wheeler (1983), and Bell (1990), among many others, show.

How subtle the issue may get can be best demonstrated by the fact that in certain instances it is possible to "reconstruct" the quantum wave function after its alleged "collapse" (Greenberger and Yasin, 1989). Thereby, not a single (quantum) bit of information should remain available from the previous "measurement." In such a scenario, it is possible to "measure" complementary observables: the price to be paid amounts to the total ignorance of the first "measurement outcome."

Recently, schemes for an "interaction-free wave function collapse" associated with "interaction-free" measurement schemes have received renewed attention (Dicke, 1981; Elitzur and Vaidman, 1993; Vaidman, 1994; Kwiat *et al.*, 1995). Compare Bohr's statement (Bohr, 1928; reprinted in Wheeler and Zurek, 1983, pp. 89, 103):

> . . . the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected . . . the impossibility of neglecting the interaction with the agency of measurement means that every observation introduces a new uncontrollable element. Indeed, it follows . . . that the measurement of the positional co-ordinates of a particle is accompanied not only by a finite change in the dynamical variables, but also the fixation of its position means a complete rupture in the causal description of its dynamical behaviour, while the determination of its momentum always implies a gap in the knowledge of its spatial propagation.

with the statement by Gabor (1961, p. 124): "No observation can be made with less than one quantum passing through the observed object." Indeed, "interaction-free" measurement schemes suggest that it is no longer necessary to assume that any measurement has to be associated with the exchange of at least one quantum of action. The situation seems to conform more to an issue raised by Landauer (1989, Section 2), "What is measurement? If it is simply information transfer, that is done all the time inside the computer, and can be done with arbitrarily little dissipation."

The "folklore" understanding of complementarity in general and the Heisenberg uncertainty relation in particular is the existence of certain (complementary) features of a quantum system which cannot be measured and predicted simultaneously with arbitrary accuracy. The first (but not last) attempt to overcome a certain vagueness in its definition (Ballentine, 1970, pp. 364–367) was undertaken by Pauli (1933), who called two classical concepts complementary "if the applicability [operationalizability] of the one (e.g., position coordinate) stands in the relation of exclusion to that [operationalizability] of the other (e.g., momentum)," in the sense that any experimental setup for measuring one object interferes destructively with any experimental setup for measuring the other object (Jammer, 1966, p. 355).

The "canonical" understanding of complementarity is expressed in Messiah (1961, p. 154):

> The description of properties of microscopic objects in classical terms requires pairs of complementary variables; the accuracy in one member of the pair cannot be improved without a corresponding loss in the accuracy of the other member. . . . It is impossible to perform measurements of position x and momentum p with uncertainties (defined by the root-mean square deviations) Δx and Δp such that the product of $\Delta x \Delta p$ is smaller than a constant unit of action $\hbar/2$.

In Prigogine's words (1980, p. 51), "the world is richer than it is possible to express in any single language."

1.2. Moore's "Gedanken" Experiments

Moore (1956) studied some experiments on finite deterministic automata in an attempt to understand what kind of conclusions about the internal conditions of a finite machine it is possible to draw from input–output experiments. To emphasize the conceptual nature of his experiments, Moore borrowed from physics the term Gedanken-experiment.

In the next section we shall present the formal notion of a finite automaton. To understand Moore's approach it is enough at this stage to say that the machines we are going to consider are *finite* in the sense that they have a finite number of states, a finite number of input symbols, and a finite number of output symbols. Such a machine has a strictly deterministic behavior: the current state of the machine depends only on its previous state and previous input; the current output depends only on the present state.

A (simple) Moore experiment can be described as follows: a copy of the machine will be experimentally observed, i.e., the experimenter will input a finite sequence of input symbols to the machine and will observe the sequence of output symbols. The correspondence between input and output symbols depends on the particular chosen machine and on its initial state. The experimenter will study the sequences of input and output symbols and will try to conclude that "the machine being experimented on was in state q at the beginning of the experiment."[5]

[5] This is often referred to as a *state identification experiment*.

Moore's experiments have been studied from a mathematical point of view by various researchers, notably Ginsburg (1958), Gill (1961), Chaitin (1965), Conway (1971), and Brauer (1984).

1.3. Finite Deterministic Automata

A finite deterministic automaton consists of a finite set of states and a set of transitions from state to state that occur on input symbols chosen from some fixed alphabet. For each symbol there is exactly one transition out of each state, possible back to the state itself. So, formally, a *finite automaton* consists of a finite set Q of states, an input alphabet Σ, and a transition function δ: $Q \times \Sigma \to Q$. Sometimes a fixed state, say 1, is considered to be the *initial state*, and a subset F of Q denotes the *final states*.

A *Moore automaton* is a finite deterministic automaton having an *output function* f: $Q \to O$, where O is a finite set of output symbols. At each time the automaton is in a given state q and is continuously emitting the output $f(q)$. The automaton remains in state q until it receives an input signal σ, when it assumes the state $\delta(q, \sigma)$ and starts emitting $f(\delta(q, \sigma))$.

In this paper we are going to almost exclusively concentrate on the case of automata on a binary alphabet $\Sigma = \{0, 1\}$ having $O = \Sigma$. So, from now on, a Moore automaton will be just a triple $M = (Q, \delta, f)$.

Let Σ^* be the set of all finite sequences (words) over the alphabet Σ, including the empty word e; by Σ^+ we denote $\Sigma^* \backslash \{e\}$. The transition function δ can be extended to a function $\bar{\delta}$: $Q \times \Sigma^* \to Q$ as follows:

$$\bar{\delta}(q, e) = q \qquad \text{for all} \quad q \in Q$$

$$\bar{\delta}(q, \sigma w) = \bar{\delta}(\delta(q, \sigma), w) \qquad \text{for all} \quad q \in Q, \quad \sigma \in \Sigma, \quad w \in \Sigma^*$$

The output produced by an experiment started in state q with input sequence $w \in \Sigma^*$ is described by $E(q, w)$, where E is the function

$$E: \quad Q \times \Sigma^* \to \Sigma^*$$

defined as follows:

$$E(q, e) = f(q)$$

$$E(q, \sigma w) = f(q)E(\delta(q, \sigma), w)), \qquad q \in Q, \quad \sigma \in \Sigma, \quad w \in \Sigma^*$$

and f: $Q \to \Sigma$ is the output function.

Consider, for example, Moore's automaton, in which $Q = \{1, 2, 3, 4\}$, $\Sigma = \{0, 1\}$. The transition is given by Table I, and the output function is defined by $f(1) = f(2) = f(3) = 0, f(4) = 1$.

The graphical representation in Fig. 1 will be consistently used in what follows.

Table I

q	σ	$\delta(q, \sigma)$	q	σ	$\delta(q, \sigma)$
1	0	4	3	0	4
1	1	3	3	1	4
2	0	1	4	0	2
2	1	3	4	1	2

The experiment starting in state 1 with input sequence 000100010 leads to the output 0100010001. Indeed,

$$E(1,000100010)$$
$$= f(1)f(4)f(2)f(1)f(3)f(4)f(2)f(1)f(3)f(4)$$
$$= 0100010001$$

Let $M = (Q, \delta, f)$ be a Moore automaton. The language generated by M on the state q is $L(M, q) = \{w \in \Sigma^* \mid f(\bar{\delta}(q, w)) = 1\}$. It is algorithmically decidable whether two languages $L(M, q)$, $L(M', q')$ are equal or not (Hopcroft and Ullman, 1979).

1.4. Reversibility

Everyone is familiar with the strange effects produced by projecting a film backward. This "strangeness" is considered to be normal in classical dynamics, as was explicitly stated by its founders such as Galileo and Huygens.[6] Quantum mechanics, in which state preparations and measurements

[6]For instance, when they described the implications of the equivalence between cause and effect as an axiom for their mathematical model of motion.

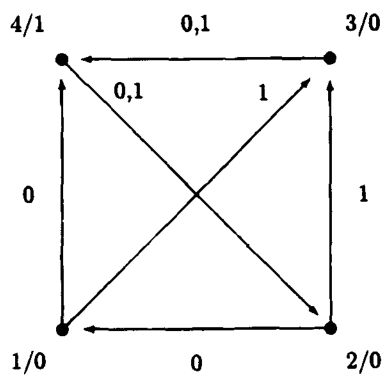

Fig. 1.

are irreversible,[7] raised serious doubts related to reversibility and initiated its abolition.[8]

A new perspective on this issue has come recently from computation theory.[9] Today's computers erase a bit of information every time they perform a computation corresponding to a many-to-one operation (Landauer, 1961; Bennett, 1973, 1982; Fredkin and Toffoli, 1982). Therefore, computational operations such as the explicit deletion of information or clearing some memory are "irreversible." In spite of the fact that in the last 50 years the dissipated energy per bit of computational operation has decreased by roughly tenfold each five years (Landauer, 1988), the erasure is done very inefficiently, and much more than kT energy is dissipated for each bit erased.[10]

In order to improve computer hardware performance we have to continue to reduce the energy dissipated by each computational operation. There are two alternative ways to approach this problem: (a) improving by conventional methods, i.e., improving the efficiency with which we erase information; (b) ultimately using computational operations that do not erase information, i.e., so-called "reversible" computational operations, which can, in principle, dissipate arbitrarily little heat.[11]

The above facts show clearly how important it is to model the idea of a reversible computational operation [for an excellent discussion see Bennett and Landauer (1984)]. In our case of finite automata, a possible definition is the following: the automaton (Q, δ, f) is *reversible* if for all states $p, q \in Q$ and $u \in \Sigma^*$ with $\bar{\delta}(p, u) = q$, there exists a word $w \in \Sigma^*$ such that $\bar{\delta}(q, w) = p$. In other words, every input state of a computation can be "reached back" from the final state of the computation by means of a suitable computation. A stronger definition has been studied in the literature (see Bavel and Muller, 1970); we will return to it later.

2. MOORE'S UNCERTAINTY REVISITED

2.1. Indistinguishability

Consider now a Moore automaton $M = (Q, \delta, f)$. Following Moore (1956), we shall say that a state q is "indistinguishable" from a state q' (with

[7] Up to instances where the wave function is reconstructed.

[8] "*Active* science is thus, by definition, extraneous to the idealized, reversible world it is describing" (Prigogine and Stengers, 1984, p. 61).

[9] More than 40 years ago Einstein had a similar point: irreversibility is an illusion, a subjective impression. *There is no irreversibility in the basic laws of physics.*

[10] Here k is Boltzmann's constant and T is the absolute temperature in Kelvin degrees, so $kT \approx 3 \times 10^{-21}$ J at room temperature.

[11] As the energy dissipated per irreversible computational operation approaches the limit of $\ln 2 \times kT$, the use of reversible operations is likely to become more attractive.

respect to M) if every experiment performed on M starting in state q produces the same outcome as it would starting in state q'. Formally,

$$E(q, x) = E(q', x)$$

for all words $x \in \Sigma^+$.

An equivalent way to express the indistinguishability of the states q and q' is to require, following Conway (1971, p. 3), that for all $w \in \Sigma^*$,

$$f(\overline{\delta}(q, w)) = f(\overline{\delta}(q', w))$$

Indeed

$$\begin{aligned} &E(q, x_1 x_2 \cdots x_n) \\ &\quad = f(q) f(\overline{\delta}(q, x_1)) f(\overline{\delta}(q, x_1 x_2)) \cdots f(\overline{\delta}(q, x_1 x_2 \cdots x_n)) \\ &\quad q \in Q, \quad x_1 x_2 \cdots x_n \in \Sigma^* \end{aligned}$$

A pair of states will be said to be "distinguishable" if they are not "indistinguishable," i.e., if there exists a string $x \in \Sigma^+$ such that $E(q, x) \neq E(q', x)$.

Moore (1956) has proven the following important theorem: *There exists a Moore automaton M such that any pair of its distinct states are distinguishable, but there is no experiment which can determine what state the machine was in at the beginning of the experiment.* He uses the automaton displayed in Fig. 1 and the argument is simple. Indeed, each pair of distinct states can be distinguished by an experiment: 1, 2 by $x = 0$; 1, 3, by $x = 1$; 1, 4 by $x = 0$; 2, 3, by $x = 0$; 2, 4, by $x = 0$; and 3, 4, by $x = 0$.

However, there is no (unique) experiment capable of distinguishing between every pair of arbitrary distinct states. Two cases have to be examined:

(A) *The experiment starts with* 1, i.e., $x = 1u$, $u \in \Sigma^*$. In this case $E(1, x) = E(2, x)$, that is, x cannot distinguish between the states 1, 2 as

$$\begin{aligned} E(1, x) = E(1, 1u) &= f(1) f(\delta(1, 1)) E(\delta(1, 1), u) \\ &= f(1) f(3) E(3, u) = 00E(3, u) \end{aligned}$$

and

$$\begin{aligned} E(2, x) = E(2, 1u) &= f(2) f(\delta(2, 1)) E(\delta(2, 1), u) \\ &= f(2) f(3) E(3, u) = 00E(3, u) \end{aligned}$$

(B) *The experiment starts with* 0, i.e., $x = 0v$, $v \in \Sigma^*$. In this case

$$E(1, x) = E(2, x)$$

that is, x cannot distinguish between the states 1, 3 as

$$\begin{aligned} E(1, x) = E(1, 0v) &= f(1)f(\delta(1, 0))E(\delta(1, 0), v) \\ &= f(1)f(4)E(4, v) = 01E(4, v) \end{aligned}$$

and

$$\begin{aligned} E(3, x) = E(3, 0v) &= f(3)f(\delta(3, 0))E(\delta(3, 0), v) \\ &= f(3)f(4)E(4, v) = 01E(4, v) \end{aligned}$$

2.2. Computational Complementarity

The difficulties in understanding and conceptualizing the complementarity phenomenon served as a motivation for considering extremely simple models featuring complementarity.[12]

Moore's theorem, described in the above section, can be thought of as being a *discrete analogue* of the Heisenberg uncertainty principle. The state of an electron E is considered specified if both its velocity and its position are known. Experiments can be performed with the aim of answering either of the following[13]:

1. What was the position of E at the beginning of the experiment?
2. What was the velocity of E at the beginning of the experiment?

For a Moore automaton, experiments can be performed with the aim of answering either of the following:

1. Was the automaton in state 1 at the beginning of the experiment?
2. Was the automaton in state 2 at the beginning of the experiment?

In either case, performing the experiment to answer question 1 changes the state of the system, so that the answer to question 2 cannot be obtained. This means that it is only possible to gain partial information about the previous

[12]This may be seen as a parallel to the Church–Turing thesis, relating the informal notion of "algorithmically computable function" to the formal term "recursive function."

[13]The propositional system obtained from the Moore automaton is the modular lattice $\mathcal{L}_{12}$ (Svozil, 1993, pp. 141–147). An exact quantum mechanical analogue has been given by Foulis and Randall (1972, Example III): Consider a device which, from time to time, emits a particle and projects it along a linear scale. We perform two experiments. In experiment A, the observer determines if there is a particle present. If there is not, the observer records the outcome of A as the outcome $\{4\}$. If there is, the observer measures its position coordinate x. If $x \geq 1$, the observer records the outcome $\{2\}$, otherwise $\{3\}$. A similar procedure applies for experiment B: If there is no particle, the observer records the outcome of B as $\{4\}$. If there is, the observer measures the x component p_x of the particle's momentum. If $p_x \geq 1$, the observer records the outcome $\{1, 2\}$, otherwise the outcome $\{1, 3\}$. Still another quantum mechanical analogue has been proposed by Giuntini (1991, pp. 159–162). A pseudoclassical analogue has been proposed by Cohen (1989) and Wright (1990).

history of the system, since performing experiments causes the system to "forget" about its past.

Moore's automaton is a simple model featuring an "uncertainty principle" (Conway, 1971, p. 21), later termed "computational complementarity" by Finkelstein and Finkelstein (1983). These types of models have been intensively studied from the point of view of their experimental logical structure by Grib and Zapatrin (1990, 1992) as well as by Svozil (1993), Schaller and Svozil (1994, 1995, 1996), and Dvurečenskij *et al.* (1995). See Svozil and Zapatrin (1996) for a comparison of models.[14]

In what follows we introduce two nonequivalent concepts of computational complementarity based upon modeling finite automata. Informally, they can be expressed as follows. Consider the class of all elements of reality[15] and consider the following properties.

- **A** Any two distinct elements of reality can be mutually distinguished by a suitably chosen measurement procedure (Bridgman, 1934).
- **B** For any element of reality, there exists a measurement which distinguishes between this element and all the others. That is, a distinction between any one of them and all the others is operational.
- **C** There exists a measurement which distinguishes between any two elements of reality. That is, a single predefined experiment operationally exists to distinguish between an arbitrary pair of elements of reality. (Classical case.)

A natural question arises: Do there exist automata having property **C**? The answer is affirmative, and Fig. 2 gives an example of such an automaton.[16]

The experiment 10 distinguishes between any pair of distinct states. An automaton having **C** but requiring a longer experiment is presented in Section 2.4.

[14] *A note of precaution.* The analogy between automaton logic and quantum logic must be understood on the level of elements of reality. That is, elements of physical reality correspond to *equivalence classes* of automaton states—they are not necessarily associated with *single* automaton states. Every equivalence class is characterized by the requirement that al automaton states contained therein respond identically with respect to a particular input–output experiment. To state the same precaution differently: It would be misleading to assume that any automaton state corresponds to a bona fide element of physical reality (though perhaps hidden). Because, whereas in models of automaton complementarity it might still be possible to pretend that initially the automaton *actually is in a single automaton state,* which we just do not know (such a state can be seen if the automaton is "screwed open"), quantum mechanically this assumption leads to a Kochen–Specker contradiction (Kochen and Specker, 1967; Peres, 1993; Mermin, 1993; Svozil and Tkadlec, 1996).

[15] The terms "elements of reality," "properties," and "observables" will be used as synonyms.

[16] Note that the automaton in Fig. 2 is connected, i.e., every two states are linked by some computation.

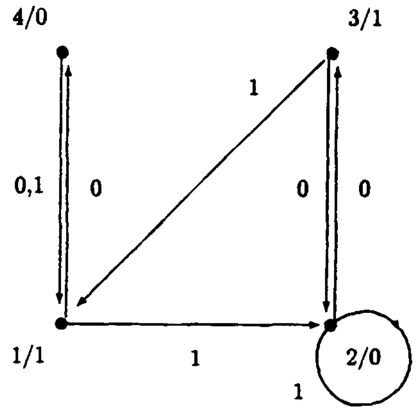

Fig. 2.

Complementarity corresponds to the following cases:

CI *Property* **A** *but not property* **B** *(and therefore not* **C***):* The elements of reality can be mutually distinguished by experiments, but one of these elements cannot be distinguished from all the others by any single experiment.

CII *Property* **B** *but not property* **C***:* Any element of reality can be distinguished from all the other ones by a single experiment, but there does not exist a single experiment which distinguishes between any pair of distinct elements.

Only the second type of complementarity deserves more attention. A Moore automaton has *CII* in case:

1. For every state q there exists an experiment w_q (depending upon q) such that $E(q, w_q) \neq E(q', w_q)$ for every state q' different from q; and
2. For every experiment w there exist at least two distinct states q, q' (depending upon w) such that $E(q, w) = E(q', w)$.

In view of condition 2, each experiment "generates" a pair of distinct states which exercise a mutual influence, namely they cannot be separated by the experiment w; this influence mimics, in a sense, the state of *quantum entanglement.*[17] To put it very pointedly, *CII* may be conceived as a *toy model* for the *EPR effect* (Einstein *et al.*, 1935; Penrose, 1990, 1994) as well as for the *Zou–Wang–Mandel effect* (Zou *et al.*, 1991; Wang *et al.*, 1991; Greenberger *et al.*, 1993). Under *CII*, for each experiment w we have at least two states q, q' [as distant as we like in terms of the emitting outputs $f(q)$, $f(q')$] which

[17] In particular, this influence cannot be used to send an actual message from one state to the other.

interact via the experiment w: any measurement of q is affecting q' and, conversely, any measurement of q' is affecting q. It is interesting to note that this explanation supports Penrose's view (1994, p. 237) that *EPR effects* are *puzzle mysteries*, that is, *genuinely puzzling, but directly experimentally supported.* Greenberger *et al.* (1993) call similar experiences simply *mindboggling*. In fact, a second "reading" of these phenomena could prove that their puzzling nature might "not be so puzzling" after all (see Section 3 for a more detailed discussion).

From a mathematical point of view properties **A**, **B**, **C** can be expressed as follows. Let $M = (Q, \delta, f)$ be a Moore automaton.

- The automaton M has property **A** if every pair of different states of M are distinguishable, i.e., for all distinct states q, q' there exists a word $w \in \Sigma^+$ (depending upon q, q') such that $E(q, w) \neq E(q', w)$.
- The automaton M has property **B** if every state M is distinguishable from any other distinct state, i.e., for every state q there exists a word $w \in \Sigma^+$ (depending upon q) such that $E(q, w) \neq E(q', w)$, for every state q' distinct from q.
- The automaton M has property **C** if there exists an experiment distinguishing between each different state of M, i.e., there exists a word $w \in \Sigma^+$ such that $E(q, w) \neq E(q', w)$, for all distinct states q, q'.

Of course, **C** implies **B**, which in turn implies **A**; none of the converse implications are true. Moore's automaton (see Fig. 1) has **A** but non-**B**. The automaton in Fig. 3 has **B** but non-**C**.

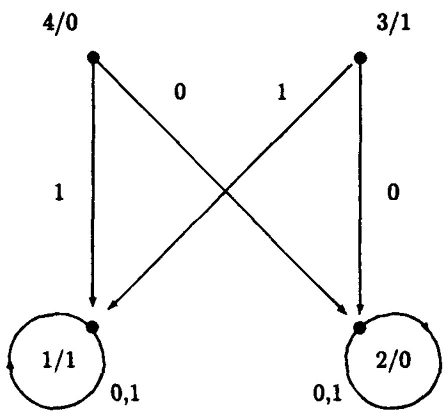

Fig. 3.

Indeed, the following pairs of states are distinguishable by every experiment: (1, 2), (1, 4), (2, 3), (3, 4). Accordingly, 1 is distinguishable from the other states by $w = 0$; 2 is distinguishable by $w = 1$; 3 is distinguishable by $w = 0$; and 4 is distinguishable by $w = 1$; so the automaton has property **B**. It does not have property **C** because:

- Any experiment w which starts with 1, i.e., $w = 1x$, $x \in \Sigma^*$, does not distinguish between 1 and 3.
- Any experiment w which starts with 0, i.e., $w = 0y$, $y \in \Sigma^*$, does not distinguish between 2 and 4.

The above example, though easy to deal with, is not very interesting (an explanation of this fact will be given in Section 2.8), as the automaton is not reversible (we can reach 4 from 1, but we cannot get 1 from 4).

Before studying further the above phenomena, we will have to deal with some natural mathematical questions: (a) is each of the properties **A**, **B**, **C** decidable? (b) How difficult is it to test these properties?

2.3. Deciding Properties A, B, C

Are properties **A**, **B**, **C** algorithmically decidable? From the work of Conway (1971, Chapter 2), it follows directly that the problem of testing whether two states are distinguishable is algorithmically decidable. This means that **A** is algorithmically decidable. In this section we will present a uniform proof for the decidability of properties **A**, **B**, **C**.[18]

Start with the automaton $M = (Q, \delta, f)$ and for each state $q \in Q$ construct the finite deterministic automaton

$$M_q = (Q \cup \{\#\}, \Sigma \times \Sigma, q, \delta', Q)$$

with initial state q and final states Q; here # is a new symbol added to Q. The transition function δ' is defined as follows: $\delta'(p, (\sigma, \tau)) = \delta(p, \sigma)$ in case $f(p) = \tau$, and $\delta'(p, (\sigma, \tau)) = \#$ otherwise. Here $p \in Q \cup \{\#\}$ and σ, $\tau \in \Sigma$.

Accordingly, for $p \in Q$ and $(\sigma_1, \tau_1)(\sigma_2, \tau_2) \cdots (\sigma_n, \tau_n) \in (\Sigma^2)^*$,

$$\overline{\delta'}(p, (\sigma_1, \tau_1)(\sigma_2, \tau_2) \cdots (\sigma_n, \tau_n)) \neq \#$$
$$\text{iff } E(p, \sigma_1\sigma_2 \cdots \sigma_{n-1}) = \tau_1\tau_2 \cdots \tau_{n-1}\tau_n$$

and in case the above condition holds true

$$\overline{\delta'}(p, (\sigma_1, \tau_1)(\sigma_2, \tau_2) \cdots (\sigma_n, \tau_n)) = \overline{\delta}(p, \sigma_1\sigma_2 \cdots \sigma_n)$$

[18] Our proof is a language-theoretic one; we have not been able to extend Conway's algebraic method to properties **B**, **C**. We will return to this question in the next section when dealing with the complexity of these decision problems.

We claim that for all states $p, q \in Q$,

$$p, q \quad \text{are distinguishable} \quad \text{iff } L(M_p) \neq L(M_q)$$

Assume first that p and q are distinguishable, that is, there exists $w \in \Sigma^+$ such that $E(p, w) \neq E(q, w)$. Let $w = \sigma_1 \cdots \sigma_n$ and $E(p, w) = \tau_1\tau_2 \cdots \tau_{n+1}$. Take an arbitrary letter $\sigma_{n+1} \in \Sigma$ and put

$$\alpha = (\sigma_1, \tau_1)(\sigma_2, \tau_2) \cdots (\sigma_n, \tau_n)(\sigma_{n+1}, \tau_{n+1})$$

Clearly, $\alpha \in L(M_p)\backslash L(M_q)$: $\overline{\delta'}(p, \alpha) = \overline{\delta}(p, w)$ $[E(p, w) = \tau_1\tau_2 \cdots \tau_{n+1}]$, but $\overline{\delta'}(q, \alpha) = \#(E(p, w) \neq E(q, w))$.

Conversely, assume that $L(M_p) \neq L(M_q)$. Then, there exists $\alpha \in L(M_p)\backslash L(M_q)$ [or $\alpha \in L(M_q)\backslash L(M_p)$]. Let $\alpha = (\sigma_1, \tau_1)(\sigma_2, \tau_2) \cdots (\sigma_n, \tau_n)$, and $w = \sigma_1\sigma_2 \cdots \sigma_{n-1}$. From hypothesis, $E(p, w) = \tau_1 \cdots \tau_n$ $[\alpha \in L(M_p)]$, but $E(q, w) \neq \tau_1 \cdots \tau_n$ $[\alpha \notin L(M_q)]$, so $E(p, w) \neq E(q, w)$, i.e., p, q are distinguishable.

We are now able to conclude that *properties* **A**, **B**, **C** *are algorithmically decidable*. Indeed, first we notice that

$$\mathbf{A} \quad \text{is true} \quad \text{iff } L(M_p) \neq L(M_q), \quad \text{for all} \quad p, q \in Q, \quad p \neq q$$

As the problem of testing whether $L(M_p) \neq L(M_q)$ is algorithmically decidable, it follows that **A** is decidable.

For **B** let M_q, $q \in Q$, be defined as above. For each $q \in Q$ define the set

$$S(q) = \bigcap_{p \in Q, p \neq q} ((L(M_q)\backslash L(M_p)) \cup (L(M_p)\backslash L(M_q)))$$

Another way of computing the sets $S(q)$ is to define

$$D_{p,q} = (L(M_p)\backslash L(M_q)) \cup (L(M_q)\backslash L(M_p))$$

for all states p, q. Then we notice the equivalence

$$\mathbf{A} \quad \text{is true} \quad \text{iff } D_{p,q} \neq \emptyset, \quad \text{for all} \quad p, q \in Q, \quad p \neq q$$

Consequently, the set $S(q)$ can be defined by

$$S(q) = \bigcap_{p \in Q, p \neq q} D_{q,p}$$

Clearly,

$$\mathbf{B} \quad \text{is true} \quad \text{iff for every} \quad q \in Q, \quad S(q) \neq \emptyset$$

Finally, the decidability of **C** follows from the formula

$$\mathbf{C} \quad \text{is true} \quad \text{iff } \bigcap_{q \in Q} S(q) \neq \emptyset$$

2.4. Complexity Issues

First we note that *complementarity properties CI, CII cannot appear for Moore automata with fewer than four states.*[19] Indeed, if $M = (Q, \delta, f)$ has fewer than two states, the statement is clear. So, let $Q = \{1, 2, 3\}$. We only need to consider two cases[20]:

1. If $f(1) = f(2) = f(3)$, then no pair of states is distinguishable.
2. If $f(1) = f(2)$ and $f(1) \neq f(3)$, and the states 1, 2 are distinguishable, then there exists an experiment $w \in \Sigma^+$ such that $E(1, w) \neq E(2, w)$. Since $f(1) = f(2) \neq f(3)$, $E(1, w) \neq E(3, w)$ and $E(2, w) \neq E(3, w)$. Accordingly, **A** is equivalent to **C**.

An interesting problem is to evaluate how difficult it is to test properties **A**, **B**, **C**. A way to look at this problem is to evaluate the shortest length of experiments needed to decide properties **A**, **B**, **C**. This problem has been studied for **A** by some authors (for instance, Chaitin, 1965; Conway, 1971). The main result (for a binary alphabet) can be stated as follows: *to test* **A**, *it is sufficient to test the condition $E(q, w) \neq E(q', w)$ for all words of length less than* $\#(Q) - 2$. In fact, it is trivial to notice that we need only test the above condition for words of length *equal* to $\#(Q) - 2$.

This result is no longer true for **B** and **C**. Indeed, 101010101 is the shortest word that distinguishes every pair of states in the automaton $M = (\{1, 2, 3, 4, 5, 6, 7\}, \delta, f)$ displayed in Fig. 4.[21]

Here is the argument. Each $w \in 1^+01^+01^+01^+01(0 + 1)^*$ *can* distinguish every pair of states in M. The shortest word in that set is $w = 101010101$. No shorter experiment can replace w[22]:

- Every word $w \in 0(0 + 1)^*$ cannot distinguish between states 5, 6.
- Every word $w \in 1^+00(0 + 1)^*$ cannot distinguish between states 4, 5.
- Every word $w \in 1^+01^+00(0 + 1)^*$ cannot distinguish between states 3, 4.
- Every word $w \in 1^+01^+01^+00(0 + 1)^*$ cannot distinguish between states 2, 3.

[19] This result was noticed by Conway (1971, pp. 20–23) for *CI*.

[20] It is worth noticing that this is true *regardless of the size of the alphabet.* Indeed, assuming that Σ has more than two symbols, the following analysis should be completed with one more case: if $f(1) \neq f(2)$, $f(2) \neq f(3)$, and $f(1) \neq f(3)$, then M has **C**.

[21] In general, the shortest word that can distinguish every pair of states in the automaton $(\{1, 2, \ldots, n\}, \delta, f)$, where $\delta(i, 0) = i + 1$, $\delta(i, 1) = i$, for $1 \leq i \leq n - 2$, $\delta(n - 1, 0) = n - 1$, $\delta(n - 1, 1) = \delta(n, 0) = \delta(n, 1) = n$, $f(i) = 0$, $1 \leq i \leq n - 1$, $f(n) = 1$ has length $2n - 5$.

[22] We next use the classical notation for regular expressions (Hopcroft and Ullman, 1979, pp. 28–29).

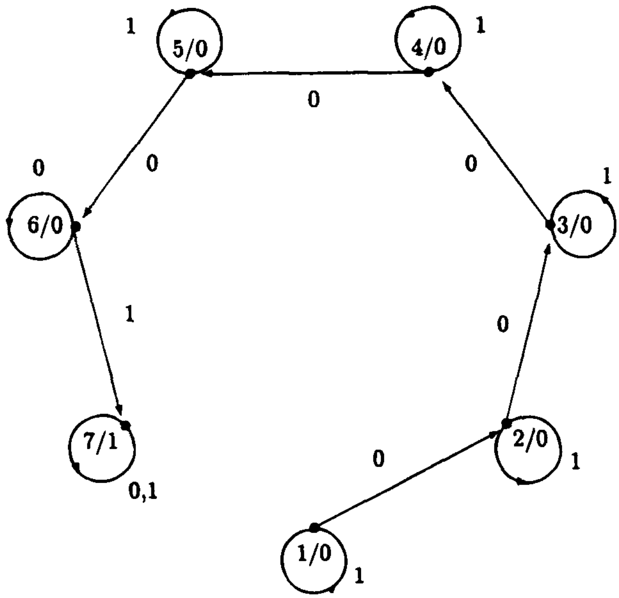

Fig. 4.

- Every word $w \in 1^+01^+01^+01^+00(0+1)^*$ cannot distinguish between states 1, 2.

2.5. Operators

A probably better physical way to look at an automaton, which is equivalent to the classical mathematical one, is to think of M in terms of the transformations (operators) T_σ: $Q \to Q$, $\sigma \in \Sigma$, $T_\sigma(q) = \delta(q, \sigma)$, as "pushbuttons" allowing the automaton to change its states. Mathematically, we shall associate to M a class of operators $(T_w)_{w \in \Sigma^*}$,

$$T_w\colon \quad Q \to Q, \quad T_w(q) = \bar{\delta}(q, w)$$

Clearly, for all $u, w \in \Sigma^*$, $T_u \circ T_v = T_{uv}$, so $(T_w)_{w \in \Sigma^*}$ is a monoid (T_e is the neutral element).[23] In fact, this monoid is finite: define the equivalence relation $u \approx v$ if $T_u = T_v$, pick from each equivalence class the smallest word (in quasilexicographic order), and collect all these words into the finite set S. Then each operator T_u has a "name" T_v with $v \in S$.

[23]This monoid is sometimes called the *transition monoid* (Clifford and Preston, 1961); for more details on the algebraic theory see also Gécseg and Peák (1972).

Sometimes the monoid of operators is a group, and in this special situation both types of complementarity disappear.[24] Here is the mathematical justification.

The following three statements are equivalent:

G1. The operators $(T_w)_{w\in S}$ form a group.

G2. Each operator T_σ, $\sigma \in \Sigma$, is bijective.[25]

G3. Every operator T_σ, $\sigma \in \Sigma$, has a right inverse.

The implications from G1 to G2 and from G2 to G3 are trivial. So, let us assume that G3 holds, i.e., for every $\sigma \in \Sigma$ there exists a word x_σ (depending upon σ) such that $T_{\sigma x_\sigma} = T_e$. We shall prove G1.

Note first that each operator has a right inverse, that is, for each word u there corresponds a word $\bar{u}$ such that $T_{u\bar{u}} = T_e$. Indeed, if $u = \sigma_1\sigma_2 \cdots \sigma_n$, then $\bar{u} = x_{\sigma_n} \cdots x_{\sigma_1}$ does the job. Now take $p = T_{\bar{u}u}(q)$. As

$$\bar{\delta}(p, \bar{u}) = \bar{\delta}(\bar{\delta}(q, \bar{u}u), \bar{u}) = \bar{\delta}(\bar{\delta}(q, \bar{u}), u\bar{u}) = \bar{\delta}(q, \bar{u})$$

it follows that

$$\bar{\delta}(p, \bar{u}) = \bar{\delta}(q, \bar{u})$$

Using again the hypothesis and the above equality, we get

$$p = \bar{\delta}(\bar{\delta}(p, \bar{u}), \bar{\bar{u}}) = \bar{\delta}(\bar{\delta}(q, \bar{u}), \bar{\bar{u}}) = q$$

which tells us that $T_{\bar{u}}$ is the inverse of T_u.

Each of the above equivalent conditions G1–G3 *implies the equivalence of* **A**, **B**, *and* **C**. Indeed, every operator T_w has an inverse $T_{\bar{w}}$. If M has **A**, then for each pair of distinct states q, q' there exists a word $w_{q,q'}$ such that $E(q, w_{q,q'}) \neq E(q', w_{q,q'})$. To get an experiment which globally distinguishes between any two distinct states, we proceed as follows: we concatenate all words $w_{q,q'}\overline{w_{q,q'}}$ (when q, q' range in Q are distinct) and we get the word w such that for all distinct states q, q', we have $E(q, w) \neq E(q', w)$.

[24] If we remove T_e from $(T_w)_{w\in\Sigma^*}$ we get a semigroup which has also a very interesting structure; for example, this semigroup may be a group even in case $(T_w)_{w\in\Sigma^*}$ is not. For the automaton [suggested to us by H. Jürgensen (private communication, 1996)] whose transitions are given by the table

q	σ	$\delta(q, \sigma)$	q	σ	$\delta(q, \sigma)$
1	0	1	3	0	3
1	1	3	3	1	1
2	0	1	4	0	4
2	1	3	4	1	4

one has $T_{01} = T_{10} = T_1$ and $T_{00} = T_{11} = T_0$. Consequently, $(T_u)_{u\in\Sigma^*\setminus\{e\}}$ is a group, but none of the generators T_0, T_1 is injective.

[25] As each operator is a function from the finite set Q into itself, it follows that an operator T_σ is bijective iff it is injective iff it is surjective.

Intuitively, the statement "G1–G3 imply the absence of complementarity" can be understood as follows. Suppose an automaton has one of the equivalent properties G1–G3. Suppose further than an observer obtains a single copy of it and adopts the following strategy. The observer runs an arbitrary number of independent experiments. After each experiment, the observer records its outcome and steers the automaton back to its original (unknown) state.[26] In this way, the observer can make sure that the experiment does not irreversibly destroy potential information about the initial state of the automaton. Indeed, the setup is similar to Moore's multi-automaton configuration, with the only difference that not all experiments are performed simultaneously, but only one at a time. In that way, total and thus classical knowledge of the initial state is obtained.

This strategy fails for quantum systems. There, it is only possible to "reverse the wave function collapse" ("reconstruct the state") if no knowledge of the measurement outcome is left over. All obtained information is needed in the reconstruction process itself. And, since any copying of proper $q(u)$ bits of quantum information is not allowed, the strategy fails to produce the classical "elements of reality." Because, stated pointedly, the observer either can make sure that he recovers the original system, (exclusive) or records a single measurement outcome associated with one particular experiment.

From an observer "from inside," the automaton is reversible, i.e., each computation can be reversed. To be more precise, an "inside observer" can reverse any computation, but the proof that *each computation can be reversed* can be achieved only at the meta-level, i.e., at the level of a language "speaking about the automaton." An external observer is "losing" information in the process of monitoring only the outcomes: for such an observer some computations cannot be reversed.

2.6. Measuring the Complexity of Automata

The *size* of the monoid of operators is a measure of the complexity of the automaton. In what follows we shall refer to this measure as the "size of the automaton."

Experimental tests for the case of four-state automata show that each automaton satisfying *CI* or *CII* has a *noncommutative* monoid of operators. In physical terms, noncommutativity is a mathematical form of complementarity, meaning that "there is no state in which both measurables have well-defined values simultaneously."

How far could one go from the monoid structure associated with automata to the unitary transformations encountered in the evolution of quantum

[26] Bennett (1973) has used a similar strategy for avoiding a huge memory overhead in reversible computations.

mechanical states between measurements? A few warnings should be issued at the very beginning. Automaton logic, like quantum logic, is *static*. It is not concerned with dynamical processes, but with the inference of operational statements and their interrelation. Also, quantum systems are defined in the entire richness of finite/infinite-dimensional Hilbert spaces and it can be expected that no finite structure can faithfully represent such a wealth of mathematics (Svozil and Tkadlec, 1996). Therefore, there is little hope for complete "isomorphism."

But can one go further and find a correspondent for unitarity? That is, what corresponds to complex conjugation * and transposition $'$ in the automaton context? As pointed out above, the composition $(U^*)'$ should correspond to the reverse transition U^{-1} (which exists when the operator monoid is a group; in such a case the automaton is reversible). There is one more issue: Unitary transformations are generated by Hermitian ones via $U(n) = \exp[itH(n)]$. Could the Hamiltonian $H(n)$ be given any meaning in the automaton context? We shall leave these questions open at the moment.

2.7. Complementarity: Reversible Instances

Recall that the automaton (Q, δ, f) was termed *reversible*[27] if for all states $p, q \in Q$ and $u \in \Sigma^*$ with $\overline{\delta}(p, u) = q$, there exists a word $w \in \Sigma^*$ such that $\overline{\delta}(q, w) = p$.

The following two conditions are each equivalent to reversibility:

1. For all $q \in Q$ and $w \in \Sigma^*$ there exists a word $u \in \Sigma^*$ such that $\overline{\delta}(q, wu) = q$.
2. For every state $q \in Q$ and $\sigma \in \Sigma$ there exists a word $v_\sigma^q \in \Sigma^*$ (depending upon σ and q) such that $\overline{\delta}(q, \sigma v_\sigma^q) = q$.

We have to prove only the equivalence between the last condition and reversibility. Indeed, if $u = \sigma_1 \cdots \sigma_n$; then

$$\overline{\delta}(q, uv_n^{\overline{\delta}(q,\sigma_1\cdots\sigma_{n-1})} \ldots v_2^{\overline{\delta}(q,\sigma_1)}v_1^q) = q$$

Each of the above conditions is decidable, as we need to examine only words of length less than the size of Q, due to the Pumping Lemma (see Hopcroft and Ullman, 1979, pp. 55–56): for all $q \in Q$, $u \in \Sigma^*$, we have $\overline{\delta}(q, u) = \overline{\delta}(q, w)$, for some $w \in \Sigma^*$ with length less than the size of Q.

Experimental computations[28] show that out of 359,040[29] reversible four-state automata: (i) 26,688 satisfy *CI*; the minimal size of an automaton

[27] Condition G3 was used as a definition for "reversibility" by Bavel and Muller (1970); it seems to us to be too strong.
[28] Sample programs can be obtained from E.C.
[29] At this stage no attempt has been made to distinguish between "isomorphic automata."

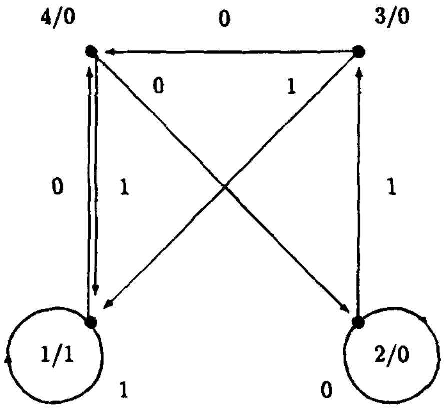

Fig. 5.

displaying *CI* is 8 (see example in Fig. 5); the maximal size is 79 (see example in Fig. 7); and (ii) 16,128 satisfy *CII*; the minimal size of an automaton displaying *CI* is 9 (see example in Fig. 6); the maximal size is 145 (see example in Fig. 8).

We start with examples of automata having *CI*, *CII* and minimal size. The automaton in Fig. 5 has *CI*. Indeed, the automaton has **A**: the pairs of states (1, 2), (1, 3), (1, 4) are distinguishable by any experiment, (2, 3) and (2, 4) can be distinguished by $w = 1$, and (3, 4) can be distinguished by $w = 01$. The automaton does not have **B**, since for 4, (i) every experiment w of the form $0y$, $y \in \Sigma^*$, does not distinguish between 2 and 4, and (ii) every experiment w of the form $1y$, $y \in \Sigma^*$, does not distinguish between 3 and 4.

The monoid of operators has the eight elements given in Table II, and the induced table is given in Table III.

The automaton in Fig. 6 satisfies *CII*. The automaton has **B**: the states (1, 2), (1, 4), (2, 3), (3, 4) are distinguishable by any experiment, so the state 1 can be distinguished from any other state by $w = 1$; 2 can be distinguished by $w = 0$; 3 can be distinguished by $w = 1$; and 4 can be distinguished by $w = 0$. It does not have **C**, since (i) if $w = 0y$, $y \in \Sigma^*$, then 1 and 3 are not distinguishable, and (ii) if $w = 1y$, $y \in \Sigma^*$, then 2 and 4 are not distinguishable.

Table II

State	T_e	T_0	T_1	T_{00}	T_{01}	T_{10}	T_{11}	T_{001}
1	1	4	1	2	1	4	1	3
2	2	2	3	2	3	4	1	3
3	3	4	1	2	1	4	1	3
4	4	2	1	2	3	4	1	3

Table III

$\circ$	T_e	T_0	T_1	T_{00}	T_{01}	T_{10}	T_{11}	T_{001}
T_e	T_e	T_0	T_1	T_{00}	T_{01}	T_{10}	T_{11}	T_{001}
T_0	T_0	T_{00}	T_{01}	T_{00}	T_{001}	T_{10}	T_{11}	T_{001}
T_1	T_1	T_{10}	T_{11}	T_{00}	T_{11}	T_{10}	T_{11}	T_{001}
T_{00}	T_{00}	T_{00}	T_{001}	T_{00}	T_{001}	T_{10}	T_{11}	T_{001}
T_{01}	T_{01}	T_{10}	T_{11}	T_{00}	T_{11}	T_{10}	T_{11}	T_{001}
T_{10}	T_{10}	T_{00}	T_{11}	T_{00}	T_{001}	T_{10}	T_{11}	T_{001}
T_{11}	T_{11}	T_{10}	T_{11}	T_{00}	T_{11}	T_{10}	T_{11}	T_{001}
T_{001}	T_{001}	T_{10}	T_{11}	T_{00}	T_{11}	T_{10}	T_{11}	T_{001}

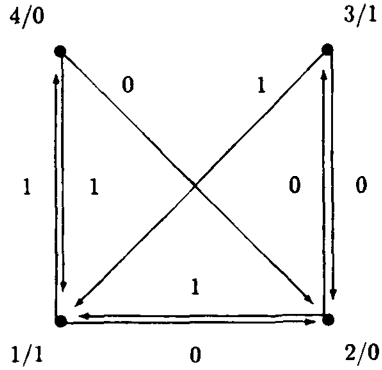

Fig. 6.

In this case the monoid of operators has the nine elements given in Table IV, and the induced table is given in Table V.

We continue with examples of automata having *CI*, *CII*, and maximal size. The automaton in Fig. 7 has *CI* and maximum size (i.e., 79).

The pairs of states (1, 2), (1, 3), (1, 4) can be distinguished by any experiment. For (2, 3) and (2, 4) we can use $w = 1$; for (3, 4) we can use $w = 0$. So, it has **A**.

The automaton does not have **B**, since (2, 4) cannot be distinguished by any experiment $w = 0y$, $y \in \Sigma^*$, and (3, 4) cannot be distinguished by any experiment $w = 1y$, $y \in \Sigma^*$.

Table IV

State	T_e	T_0	T_1	T_{00}	T_{01}	T_{10}	T_{11}	T_{001}	T_{100}
1	1	2	4	3	1	2	1	4	3
2	2	3	1	2	1	2	4	4	3
3	3	2	1	3	1	2	4	4	3
4	4	2	1	3	1	2	4	4	3

Table V

$\circ$	T_e	T_0	T_1	T_{00}	T_{01}	T_{10}	T_{11}	T_{011}	T_{100}
T_e	T_e	T_0	T_1	T_{00}	T_{01}	T_{10}	T_{11}	T_{011}	T_{100}
T_0	T_0	T_{00}	T_{01}	T_0	T_{01}	T_{10}	T_{011}	T_{011}	T_{100}
T_1	T_1	T_{10}	T_{11}	T_{100}	T_{01}	T_{10}	T_1	T_{011}	T_{100}
T_{00}	T_{00}	T_0	T_{01}	T_{00}	T_{01}	T_{10}	T_{011}	T_{011}	T_{100}
T_{01}	T_{01}	T_{10}	T_{011}	T_{100}	T_{01}	T_{10}	T_{01}	T_{011}	T_{100}
T_{10}	T_{10}	T_{100}	T_{01}	T_{10}	T_{01}	T_{10}	T_{011}	T_{011}	T_{100}
T_{11}	T_{11}	T_{10}	T_1	T_{100}	T_{01}	T_{10}	T_{11}	T_{011}	T_{100}
T_{011}	T_{011}	T_{10}	T_{01}	T_{100}	T_{01}	T_{10}	T_{011}	T_{011}	T_{100}
T_{100}	T_{100}	T_{10}	T_{01}	T_{100}	T_{01}	T_{10}	T_{011}	T_{011}	T_{100}

The automaton in Fig. 8 has *CII* maximum size (i.e., 145). The pair of states (1, 2), (2, 3), (2, 4) can be distinguished by any experiment. The state 1 can be distinguished by any other state by $w = 1$; 2 can be distinguished by any experiment; 3 can be distinguished by $w = 001$; and 4 can be distinguished by $w = 01$. So, it has **B**.

The automaton does not have **C**, since: (i) (3, 4) cannot be distinguished by any experiment $w = 1y$, $y \in \Sigma^*$; (ii) if $w = 0^n$, for some $n \geq 1$, then 1 and 3 cannot be distinguished: $E(1, O^n) = 0^{n+1} = E(3, O^n)$; (iii) if $w = 0^{3n}1y$, for some $y \in \Sigma^*$, $n \geq 1$, then 3 and 4 cannot be distinguished: $E(1, O^{3n}1) = 0^{3n+2} = E(4, O^{3n}1)$ and

$$\delta(3, 0^{3n}1y) = \bar{\delta}(3, 1y) = \bar{\delta}(1, y) = \bar{\delta}(4, 1y) = \bar{\delta}(4, 0^{3n}1y)$$

(iv) if $w = 0^{3n}01y$, for some $y \in \Sigma^*$, $n \geq 1$, then 1 and 3 cannot be distinguished: $E(1, O^{3n}01) = 0^{3n+3} = E(4, O^{3n}1)$ and

$$\bar{\delta}(1, 0^{3n}01y) = \bar{\delta}(1, 01y) = \bar{\delta}(1, y) = \bar{\delta}(3, 01y) = \bar{\delta}(3, 0^{3n}01y)$$

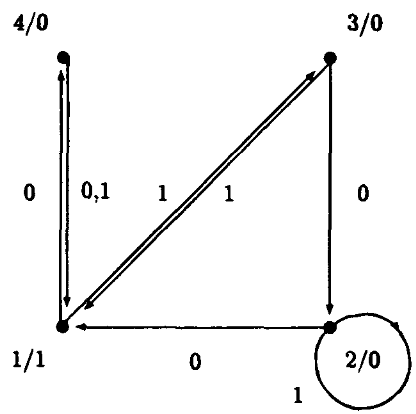

Fig. 7.

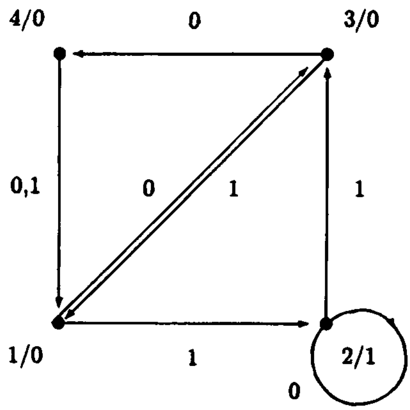

Fig. 8.

(v) if $w = 0^{3n}001y$, for some $y \in \Sigma^*$, $n \geq 1$, then 1 and 4 cannot be distinguished: $E(1, 0^{3n}001) = 0^{3n+4} = E(1, 0^{3n}001)$ and

$$\bar{\delta}(1, 0^{3n}001y) = \bar{\delta}(1, 001y) = \bar{\delta}(1, y) = \bar{\delta}(4, 001y) = \bar{\delta}(4, 0^{3n}001y)$$

2.8. More About Moore's Example

The size of Moore's automaton is 64. This automaton can be used with different output functions to produce both *CI* and *CII*. The complement of the original output function leads to *CI*. To get *CII* we can use each of the output functions in Fig. 9.

Let us prove *CII* for the first choice of *f*. Moore's automaton has **B**, since for 1 we can use $w = 01$, for 2 we can use $w = 001$, for 4 we can use $w = 1$ (any w is good for 3). The automaton does not have **C**, since: (i) (1, 2) cannot be distinguished by any experiment $w = 1y$, $y \in \Sigma^*$; (ii) if

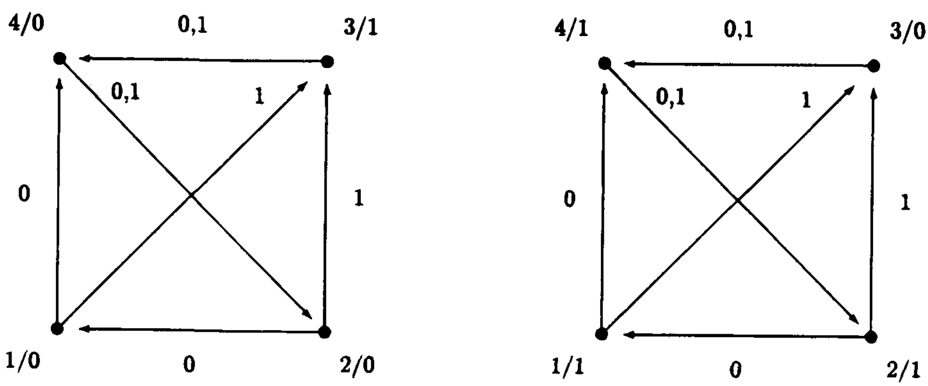

Fig. 9.

$w = 0^n$, for some $n \geq 1$, then 1 and 2 cannot be distinguished: $E(1, O^n) = 0^{n+1} = E(2, O^n)$,

$$\bar{\delta}(1, 0) = 4, \qquad \bar{\delta}(1, 00) = 2, \qquad \bar{\delta}(1, 000) = 1$$

$$\bar{\delta}(2, 0) = 1, \qquad \bar{\delta}(2, 00) = 4, \qquad \bar{\delta}(2, 000) = 2$$

(iii) if $w = 0^{3n}1y$, for some $y \in \Sigma^*$, $n \geq 1$, then 1 and 2 cannot be distinguished: $E(1, O^{3n}1) = 0^{3n+2} = E(2, O^{3n}1)$ and

$$\bar{\delta}(1, 0^{3n}1y) = \bar{\delta}(3, 1y) = \bar{\delta}(3, y) = \bar{\delta}(2, 1y) = \bar{\delta}(2, 0^{3n}1y)$$

(iv) if $w = 0^{3n}01y$, for some $y \in \Sigma^*$, $n \geq 1$, then 2 an 4 cannot be distinguished: $E(2, O^{3n}01) = 0^{3n+3} = E(4, O^{3n}01)$ and

$$\bar{\delta}(2, 0^{3n}01y) = \bar{\delta}(2, 01y) = \bar{\delta}(3, y) = \bar{\delta}(4, 01y) = \bar{\delta}(4, 0^{3n}01y)$$

(v) if $w = 0^{3n}001y$, for some $y \in \Sigma^*$, $n \geq 1$, then 1 and 4 cannot be distinguished: $E(1, O^{3n}001) = 0^{3n+4} = E(1, O^{3n}001)$ and

$$\bar{\delta}(1, 0^{3n}001y) = \bar{\delta}(1, 001y) = \bar{\delta}(3, y) = \bar{\delta}(4, 001y) = \bar{\delta}(4, 0^{3n}001y)$$

2.9. Complementarity: Nonreversible Instances

In the case of nonreversible automata the sizes of automata having *CI* or *CII* decrease, in the minimal cases, to 3. Here are two examples.

The automaton in Fig. 10 has *CI*. Indeed, the automaton has **A**: the pairs (1, 2), (1, 3), (1, 4) are distinguishable by any experiment; the word $w = 1$ distinguishes between 2, 3, and 2, 4, while $w = 0$ distinguishes between 3 and 4. The automaton does not have **B**, since the state 3 cannot be distinguished from 2 by any experiment starting with 0, and 3 cannot be distinguished from 4 by any experiment starting with 1.

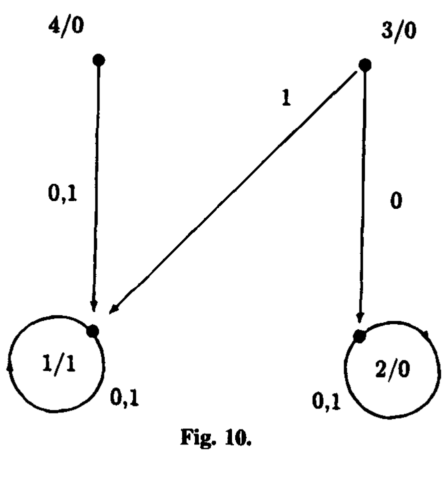

Fig. 10.

Table VI

State	T_e	T_0	T_1
1	1	1	1
2	2	2	2
3	3	2	1
4	4	1	1

$\circ$	T_e	T_0	T_1
T_e	T_e	T_0	T_1
T_0	T_0	T_0	T_0
T_1	T_1	T_1	T_1

The operators induced by the automaton in Fig. 10 and their composition table are presented in Table VI.

The automaton in Fig. 3 has *CII* (see the proof in Section 2.2). This automaton has minimum size, i.e., 3, which makes the proof for non-C quite easy. In this case we have three operators (T_e, T_0, T_1) which are different from the operators of the automaton from Fig. 7 (Table VII), but their composition tables do coincide.

3. MORE ABOUT ACTORS AND SPECTATORS

The new type of complementarity, namely *CII*, introduced in this paper mimics in a sense the state of *quantum entanglement* and may be conceived as a *toy model* for the *EPR effect* or the *Zou–Wang–Mandel effect* (Zou *et al.*, 1991; Wang *et al.*, 1991; Greenberger *et al.*, 1993). Being experimentally testable, *CII* falls into the class of *puzzle mysteries* (see Penrose, 1994, p. 237).

In fact, we believe that a bit of the mystery associated with *EPR effects* in general and with *CII* in particular might not be so "puzzling" at all, and here are two arguments.

A random (binary) sequence in the Chaitin–Martin-Löf sense (Chaitin, 1987, 1990; Calude, 1994) is the prototype of the ideal chaotic sequence in which no prediction can possibly come true and no computation can be

Table VII

State	T_e	T_0	T_1
1	1	1	1
2	2	2	2
3	3	2	1
4	4	2	1

successful in approximating more than a finite part of the sequence. However, such a sequence satisfies some very interesting *deterministic* laws. Here are two examples:

- A constructive property: each random sequence is Borel normal, in the sense that every word of length n over the binary alphabet occurs in the sequence with the exact expected probability, i.e., 2^{-n} (Calude, 1994).
- A nonconstructive property: in each random sequence at least one of the two symbols 0 and 1 must occur in arithmetic progressions of every length (van der Waerden, 1927).

With reference to normality, in a random sequence each bit has, in the long run, a "mysterious" purely deterministic influence on all digits. This "effect" can be tested (of course, only on finite initial segments of the sequence); it can be proven "from outside," i.e., at the level of the meta-language, and it is "unreachable" for any observer from "inside."

A language-theoretic version of the *EPR effect* is related to the so-called *depth hypothesis.* Psychologists have measured the "span of immediate memory," i.e. the ability to memorize at a glance and repeat correctly random digits, nonsense words, various items. It seems that the average ability is about seven (Miller, 1956). The *depth hypothesis* suggests that much of the syntactic complexity of a natural language can be understood in terms of this memory restriction.[30] From a mathematical point of view this restriction can be modeled by the property of *projectivity* (Marcus, 1967, Chapter 6), which, in a sense, measures the "long-run" syntactic subordination of words in natural languages. Again, the depth hypothesis, and more generally any syntactic subordination, is "visible" from "outside" and not from "inside."

4. CONCLUSIONS AND FURTHER WORK

Building on Moore's work concerning Gedanken-experiments, a new type of computational complementarity which mimics the state of quantum entanglement was introduced and contrasted with Moore's computational complementarity and physical complementarity. Many problems remain for further work. We mention here only a few of them: (1) Find better upper bounds for testing properties **B**, **C**; (2) describe *CI*, *CII* for Mealy automata and for nondeterministic finite automata; (3) distinguish between *isomorphic automata*; (4) investigate the influence of the size of the underlying alphabet; (5) explore the relations between *CI*, *CII*, and automata/quantum logics.

[30]The syntax of English, for instance, has many devices for keeping utterances within the bounds of this restriction; it also has resources to circumvent it, so as to regain the loss of expressive power.

ACKNOWLEDGMENTS

We express our gratitude to Prof. H. Jürgensen, R. Nicolescu, B. Pavlov, and S. Rudeanu for their comments and suggestions. This paper was completed during the visits of C.C. at the University of Technology Vienna and the University of Western Ontario (1995) and of K.S. and S.Y. at the University of Auckland in 1995–1996. C.C. has been partially supported by AURC A18/XXXXX/62090/F3414044, 1995. K.S. has been partially supported by a travel grant from the Ausseninstitut of the University of Technology Vienna.

REFERENCES

Ballentine, L. E. (1970). The statistical interpretation of quantum mechanics, *Reviews of Modern Physics*, **42**, 358–381.

Bavel, Z., and Muller, D. E. (1970). *Journal of the Association for Computing Machinery*, **17**, 231–240.

Bell, J. S. (1990). Against "measurement," *Physics World*, **3**(8), 33–40 [Reprinted in Bell (1992)].

Bell, J. S. (1992). Against "measurement," *Physikalische Blätter*, **48**, 4.

Bennett, C. H. (1973). Logical reversibility of computation, *IBM Journal of Research and Development*, **17**, 525–532 [Reprinted in Leff and Rex (1990), pp. 197–204].

Bennett, C. H. (1982). The thermodynamics of computation—A review, *International Journal of Theoretical Physics*, **21**, 905–940 [Reprinted in Leff and Rex (1990), pp. 213–248].

Bennett, C. H., and Landauer, R. (1985). The fundamental limits of computation, *Scientific American*, **1985**(July), 48–56.

Bohr, N. (1928). The quantum postulate and the recent development of atomistic theory, *Nature* **121**, 580–590 [Reprinted in Bohr (1961), pp. 52–91, and in Wheeler and Zurek (1983), pp. 87–126].

Bohr, N. (1961). *Atomic Theory and the Description of Nature*, Cambridge University Press, Cambridge.

Brauer, W. (1984). *Automatentheorie*, Teubner, Stuttgart.

Bridgman, P. W. (1934). A physicists second reaction to Mengenlehre, *Scripta Mathematica*, **2**, 101–117, 224–234.

Calude, C. (1994). *Information and Randomness—An Algorithmic Perspective*, Springer, Berlin.

Chaitin, G. J. (1965). An improvement on a theorem by E. F. Moore, *IEEE Transactions on Electronic Computers*, **EC-14**, 466–467.

Chaitin, G. J. (1987). *Algorithmic Information Theory*, Cambridge University Press, Cambridge.

Chaitin, G. J. (1990). *Information, Randomness and Incompleteness*, 2nd ed., World Scientific, Singapore.

Clifford, A. H., and Preston, G. B. (1961). *The Algebraic Theory of Semigroups*, Vols. 1 and 2, American Mathematical Society, Providence, Rhode Island.

Cohen, D. W. (1989). *An Introduction to Hilbert Space and Quantum Logic*, Springer, New York.

Conway, J. H. (1971). *Regular Algebra and Finite Machines*, Chapman and Hall, London.

Dicke, R. H. (1981). Interaction-free quantum measurements: A paradox? *American Journal of Physics*, **49**(10), 925–930.

Dvurečenskij, A., Pulmannová, S., and Svozil, K. (1995). Partition logics, orthoalgebras and automata, *Helvetica Physica Acta*, **68**, 407–428.

Einstein, A., Podolsky, B., and Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review,* **47**, 777–780 [Reprinted in Wheeler and Zurek (1983), pp. 138–141].

Elitzur, A. C., and Vaidman, L. (1993). Quantum mechanical interaction-free measurements, *Foundations of Physics,* **23**, 987–997.

Finkelstein, D., and Finkelstein, S. R. (1983). Computational complementarity, *International Journal of Theoretical Physics,* **22**, 753–779.

Foulis, D. J., and Randall, C. (1972). Operational statistics. i. Basic concepts, *Journal of Mathematical Physics,* **13**, 1667–1675.

Fredkin, E., and Toffoli, T. (1982). Conservative logic, *International Journal of Theoretical Physics,* **21**, 219–253.

Gabor, D. (1961). Light and information, *Progress in Optics,* **1**, 111–153.

Gécseg, F., and Peák, I. (1972). *Algebraic Theory of Automata,* Akademiai Kiado, Budapest.

Gill, A. (1961). State-identification experiments in finite automata, *Information and Control,* **4**, 132–154.

Ginsburg, S. (1958). On the length of the smallest uniform experiment which distinguishes the terminal states of the machine, *Journal of the Association for Computing Machinery,* **5**, 266–280.

Giuntini, R. (1991). *Quantum Logic and Hidden Variables,* BI Wissenschaftsverlag, Mannheim.

Greenberger, D. B., and Yasin, A. (1989). "Haunted" measurements in quantum theory, *Foundations of Physics,* **19**, 679–704.

Greenberger, D. B., Horne, M., and Zeilinger, A. (1993). Multiparticle interferometry and the superposition principle, *Physics Today,* **46**(August), 22–29.

Grib, A. A., and Zapatrin, R. R. (1990). Automata stimulating quantum logics, *International Journal of Theoretical Physics,* **29**, 113–123.

Grib, A. A., and Zapatrin, R. R. (1992). Macroscopic realization of quantum logics, *International Journal of Theoretical Physics,* **31**, 1669–1687.

Hopcroft, J. E., and Ullman, J. D. (1979). *Introduction to Automata Theory, Languages, and Computation,* Addison-Wesley, Reading, Massachusetts.

Jammer, M. (1966). *The Conceptual Development of Quantum Mechanics,* McGraw-Hill, New York.

Jammer, M. (1974). *The Philosophy of Quantum Mechanics,* Wiley, New York.

Kochen, S., and Specker, E. P. (1967). The problem of hidden variables in quantum mechanics, *Journal of Mathematics and Mechanics,* **17**, 59–87 [Reprinted in Specker (1990), pp. 235–263].

Kwiat, P., Weinfurter, H., Herzog, T., Zeilinger, A., and Kasevich, M. A. (1995). Interaction-free measurement, *Physical Review Letters,* **74**, 4763–4766.

Landauer, R. (1961). Irreversibility and heat generation in the computing process, *IBM Journal of Research and Development,* **5**, pp. 183–191 [Reprinted in Leff and Rex (1990), pp. 188–1965].

Landauer, R. (1988). Dissipation and noise immunity in computation and communication, *Nature,* **335**, 779–784.

Landauer, R. (1989). Computation, measurement, communication and energy dissipation, in *Selected Topics in Signal Processing,* S. Haykin, ed., Prentice-Hall, Englewood Cliffs, New Jersey, p. 18.

Landauer, R. (1994). Advertisement for a paper I like, in *On Limits,* J. L. Casti and J. F. Traub, eds., Santa Fe Institute Report 94-10-056, Santa Fe, New Mexico, p. 39.

Leff, H. S., and Rex, A. F. (1990). *Maxwell's Demon,* Princeton University Press, Princeton, New Jersey.

Marcus, S. (1967). *Algebraic Linguistics; Analytical Models,* Academic Press, New York.

Mermin, N D. (1993). Hidden variables and the two theorems of John Bell, *Reviews of Modern Physics*, **65**, 803–815.

Messiah, A. (1961). *Quantum Mechanics*, Vol. I, North-Holland, Amsterdam.

Miller, G. A. (1956). The magical number seven, plus or minus two: Social limits on our capacity for processing information, *Psychological Review*, **63**, 81–97.

Moore, E. F. (1956). Gedanken-experiments on sequential machines, in *Automata Studies*, C. E. Shannon and J. McCarthy, eds., Princeton University Press, Princeton, New Jersey.

Pauli, W. (1933). Die allgemeinen Prinzipien der Wellenmechanik, in *Handbuch der Physik*, Vol. 24, H. Geiger and K. Scheel, eds., Springer, Berlin, p. 126 [English translation in Pauli (1958), pp. 45–46 and in Pauli (1964), pp. 771–938].

Pauli, W. (1958). Principles of quantum theory, in *Encyclopedia of Physics*, Vol. 5, S. Flügge, ed., Springer, Berlin, pp. 45–46.

Pauli, W. (1964). Collected Scientific Papers, Vol. I, R. Kronig and V. F. Weisskopf, eds., Interscience, New York.

Penrose, R. (1990). *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*, Oxford University Press, Oxford.

Penrose, R. (1994). *Shadows of the Minds, A Search for the Missing Science of Consciousness*, Oxford University Press, Oxford.

Peres, A. (1993). *Quantum Theory: Concepts and Methods*, Kluwer, Dordrecht.

Prigogine, I. (1980). *From Being to Becoming*, Freeman, San Francisco.

Prigogine, I., and Stengers, I. (1984). *Order out of Chaos*, Bantam Books, Toronto.

Rössler, O. E. (1987). Endophysics, in *Real Brains, Artificial Minds*, J. L. Casti and A. Karlquist, eds., North-Holland, New York, p. 25.

Rössler, O. E. (1992). *Endophysics, Die Welt des inneren Beobachters*, Merwe Verlag, Berlin.

Schaller, M., and Svozil, K. (1994). Partition logics of automata, *Nuovo Cimento*, **109B**, 167–176.

Schaller, M., and Svozil, K. (1995). Automaton partition logic versus quantum logic, *International Journal of Theoretical Physics 34*, **8**, 1741–1750.

Schaller, M., and Svozil, K. (1996). Automaton logic, *International Journal of Theoretical Physics*, **35**, 911–940.

Specker, E. (1990). *Selecta*, Birkhäuser Verlag, Basel.

Svozil, K. (1986a). Connections between deviations from Lorentz transformation and relativistic energy-momentum relation, *Europhysics Letters*, **2**, 83–85.

Svozil, K. (1986b). Operational perception of space-time coordinates in a quantum medium, *Nuovo Cimento*, **96B**, 127–139.

Svozil, K. (1993). *Randomness and Undecidability in Physics*, World Scientific, Singapore.

Svozil, K. (1994). Extrinsic–intrinsic concept and complementarity, in *Inside versus Outside*, H. Atmanspacker and G. J. Dalenoort, eds., Springer-Verlag, Berlin, pp. 273–288.

Svozil, K., and Tkadlec, J. (1996). Greechie diagrams, nonexistence of measures in quantum logics and Kochen–Specker type constructions, *Journal of Mathematical Physics*, **37**, 5380–5401.

Svozil, K., and Zapatrin, R. R. (1996). Empirical logic of finite automata: Microstatements versus macrostatements, *International Journal of Theoretical Physics*, **35**, 1541–1548.

Toffoli, T. (1978). The role of the observer in uniform systems, in *Applied General Systems Research*, G. Klir, ed., Plenum Press, New York.

Vaidman, L. (1994). On the realization of interaction-free measurements, *Quantum Optics*, **6**, 119–126.

Van der Waerden, B. L. (1927). Beweis einer baudet'schen vermutung, *Nieuw Arkiv Wisk*, **15**, 212–216.

Wang, L. J., Zou, X. Y., and Mandel, L. (1991). Induced coherence without induced emission, *Physical Review A*, **44**, 4614–4622.

Wheeler, J. A. (1983). Law without law, in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek, eds., Princeton University Press, Princeton, New Jersey, pp. 182–213.

Wheeler, J. A., and Zurek, W. H. (1983). *Quantum Theory and Measurement*, Princeton University Press, Princeton, New Jersey.

Wigner, E. P. (1961). Remarks on the mind–body question, in *The Scientist Speculates*, I. J. Good, ed., Heinemann and Basic Books, London and New York, pp. 284–302 [Reprinted in Wheeler and Zurek (1983), pp. 168–181].

Wright, R. (1990). Generalized urn models, *Foundations of Physics*, **20**, 881–903.

Zou, X. Y., Wang, L. J., and Mandel, L. (1991). Induced coherence and indistinguishability in optical interference, *Physical Review Letters*, **67**, 318–321.